CH. 6: MOUNTAIN WHERE RAIN NEVER FALLS

Use everything you know.

Their sixth day of hiking brought the teacher and the student to an empty hut by a mountain stream. “We will rest here a while, and wash our clothes,” the teacher said.

When they had laid their clean clothes on sunny rocks to dry, the student pointed to the clouds gathering in the valley below. “Looks like rain. Should we be worried?”

“The rains have reached this place only once in the last 100 years,” the teacher said. “What is the probability that they will reach us today?”

The student thought for a moment. “Let’s say 30 storms reach the valley every year. Over 100 years, that’s 3,000 storms. But only one of them has reached the mountaintop here. So the probability is roughly 1 in 3000.”

“A very low probability,” the teacher nodded. Then she moved some of her half-dried clothes from the rocks to the inside of the hut. The student wondered why.

Later, the teacher plucked a blade of grass. “There is no dew,” she said. “This occurs less than once each month. What is the probability the rains reach us *now*?”

“Well, there’s no dew roughly 10 times per year,” the student said. “In the last century, that’s 1000 times. And at least 999 of those times, it didn’t rain. So the chances of rain are still at most 1 in 1000.” The teacher nodded, but moved more clothes inside nevertheless.

Later, a large brown bird flew directly overhead, barely 50 feet up. “The hawk-eagle flies low, just above the treetops,” the teacher said. “I glimpse such a sight only once a year. What is the probability the rain reaches us *now*?”

“Still small,” the student said. “The hawk-eagle flies low once a year, but it only rains once every century. So even if this is a sign of rain, the probability is still at most 1 in 100.” The teacher moved still more of her clothes inside the hut.

Later, as clouds began to darken the sky, the teacher pointed west, where pale stripes of color stood out against the gray sky. “A rainbow occurs in the West only once every 5 years,” the teacher said. “What is the probability that the rains reach us *now*?”

“Still low!” the student said. “There’s been a rainbow in the west 20 times in the last century. But it’s only rained one of those times. The probability is still just 1 in 20. There’s nothing to worry about.” Nevertheless, the teacher moved the last of her clothes inside the hut.

Just then, a light rain began to fall. The student scrambled to gather her clothes, but within minutes, the downpour was torrential, and everything was soaked through.

“I should have known better,” the student sighed from the shelter of the hut. “You tricked me again.”

“No trick,” the teacher said. “I told you everything.”

“How can that be?” the student said. “The odds should never have gotten higher than 1 in 20. What did you leave out? I used all three signs.”

“No.” The teacher shook her head. “You used one sign, then a different sign, then a third sign. You never used them all. How many days do you think *all three* signs have occurred?

The student blinked. “I don’t know.”

“To my knowledge,” the teacher said, “only twice: today, and once many decades ago, when I was a young girl.”

“And you still remember that day?”

“Of course,” the teacher said. “It was the day that rain reached this place.”

CH. 6: DISCUSSION

The focus here is *conditional* probability—the probability that one event will occur, given that another has already occurred.

In some sense, *all* probability is conditional. When we ask "What's the chance of rain tomorrow?" we're really asking, "What's the chance of rain tomorrow, given the conditions today?" When we ask, "What's the probability that the Patriots win the Super Bowl" we're leaving out the rest of the question: "given their success so far this season?" To compute such probabilities, you've got to know what information to use, and how to use it.

The student in this parable deserves credit for applying the first key rule: *Finding out new information may change your probability*.

For example, say we're rolling a standard die. What's the probability we roll a 4? Simple: there are six possibilities, so it's 1/6.

But what's the probability we roll a 4 *given that our roll is even*? Now we've got new information, and it changes our outlook. We can toss out 1, 3, and 5 as possibilities. That leaves 2, 4, and 6. So there's a 1/3 chance that we rolled a 4.

Or, try this: What's the probability we roll a 4 *given that our roll is a perfect square*? Well, there are only two perfect squares on a die—1 and 4. So the probability is ½.

Finally: What's the probability we roll a 4 given that our roll is even *and* a perfect square? There's only one possibility fitting this description—4 itself. So the probability is 100%. It's this final step that tripped up the student. She used one piece of information. Then she used the others. But she never used them all at once.

CH. 6: QUESTIONS

1. Some of our idioms for "That'll never happen!" can be reinterpreted as statements about conditional probability. Take your favorite low-probability event, and call it X. Then explain what each phrase has to do with conditional probability.
    a. "If X happens, then pigs fly."
    b. "X happening? That'll be the day hell freezes over."

2. We say that A and B are *dependent* if knowing that one has occurred changes the probability of the other occurring. For example, when you flip one coin, getting heads and getting tails are *dependent*. Let's see why.
    a. What's the probability that a coin flip comes up heads?
    b. What's the probability that a coin flip comes up heads, *given that it's tails*? (Don't overthink it.)
    c. Why, then are "getting heads" and "getting tails" on a coin flip dependent?

3. Meanwhile, A and B are *independent* if knowing that one has occurred *doesn't* change the probability of the other. For example, suppose we roll two dice, and we're wondering two things: (A) Was the first die a 3? and (B) Was the sum of the dice 7? These might seem like dependent events, but they're not. Let's see why.
    a. What's the probability that the sum of two dice is 7?
    b. What's the probability that the sum of two dice is 7, *given that the first die is 3*?
    c. Why, then, are "first die is a 3" and "sum of dice is 7" independent events?

4. Now, for each pair of events A and B, say whether they're dependent or independent:
    a. I pull one card from a deck. A = "the card is a diamond." B = "the card is red."
    b. I pull one card from a deck. A = "the card is a diamond." B = "the card is black."
    c. I pull two cards from a deck. A = "both cards are the same suit." B = "the first card is a diamond."
    d. I pull two cards from a deck. A = "both cards are the same suit." B = "both cards are the same number."

5. Some people define "independent events" as "unrelated events." It's not a terrible shortcut, but it's a little sloppy. Let's see why.
    a. Give an example of independent events that aren't quite "unrelated."
    b. Give an example of "unrelated" events that might not be independent. (*Note:* This obviously hinges a little on your definition of "unrelated.")